Received: *July 03, 2000,* Accepted: *October 07, 2000*
Revised: *September 26, 2000*

U(1) gauge invariance from open string field theory

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Abstract: The naive low energy effective action of the tachyon and the U(1) gauge field obtained from string field theory does not correspond to the world volume action of unstable branes in bosonic string theory. We show that there exists a field redefinition which relates the gauge field and the tachyon of the string field theory action to the fields in the world volume action of unstable branes. We identify a string gauge symmetry which corresponds to the U(1) gauge transformation. This is done to the first non linear order in the fields. We examine the vector fluctuations at the tachyon condensate till level (4,8).

Keywords: Bosonic Strings, D-branes, Conformal Field Models in String Theory.

JHEP10(2000)017

Contents

1. Introduction

Recent works have shown that string field theory is useful to understand the physics of tachyon condensation [1]. In bosonic string theory, the decay of the D 25-brane has been shown to correspond to the rolling of the tachyon of open bosonic string theory down to a new stationary point in the tachyon potential. The analysis of the tachyon potential was carried out using string field theory. It was also seen that level truncation is a good approximation scheme for the calculation of the value of the minimum of the tachyon potential and the physics of solitons in the potential [2]–[8]. The level truncation seems to be a good approximation scheme in superstring field theory [9]–[12].

As the D 25-brane has completely decayed at the new stationary point of the tachyon potential, there should be no dynamical open string degrees of freedom. In particular the U(1) gauge field should no longer be dynamical. It was conjectured in [1] that at this stationary point the effective action does not have a kinetic term for the U(1) gauge field. In fact the term multiplying the kinetic term was argued to be the tachyon potential [13]. This would render this field auxiliary. It was also suggested it would be interesting to see this phenomenon directly from string field theory. The difficulty with the direct evaluation of the coefficient of the kinetic term of the gauge field using string field theory is the problem of field redefinition.

We will now state this problem. Consider eliminating all the massive modes and the auxiliary fields of the string field using the equations of motion. We obtain the effective action $\mathcal{S}(\tilde{A}_\mu, \tilde{\phi})$ as a function of the U(1) gauge field and the tachyon $\tilde{\phi}$ at the tree level. Can we identify the gauge field $\tilde{A}_\mu$ and the tachyon $\tilde{\phi}$ with that of the gauge field of the D25-brane and its tachyon, and thus obtain the effective action of the D25-brane from string field theory? A string field Φ has the following gauge transformation in string field theory

$$\delta\Phi = Q_B\Lambda + \Lambda * \Psi - \Psi * \Lambda\,. \tag{1.1}$$

Here Λ, the infinitesimal gauge parameter is a string field of ghost number zero and "$*$" is the star product in string field theory. Let us call the U(1) field present in the expansion of Φ as $\tilde{A}_\mu$. Then, it is clear that $\tilde{A}_\mu$ does not transform as

$$\delta\tilde{A}_\mu = \partial_\mu\epsilon\,. \tag{1.2}$$

The transformation property of $\tilde{A}$ has extra terms arising from the non-abelian like transformation property of the string field Φ. Therefore the gauge field that appears in the low energy effective lagrangian A_μ which has the conventional gauge transformation property cannot be identified with $\tilde{A}_\mu$. Thus the naive low energy effective action $\mathcal{S}(\tilde{A}_\mu, \tilde{\phi})$ cannot be identified with the world volume action of the unstable brane.

It must be possible to find a relationship between A_μ and $\tilde{A}_\mu$ and ϕ and $\tilde{\phi}$. This will involve a field redefinition

$$\tilde{A}_\mu = \tilde{A}_\mu(A_\mu, \phi)\,, \qquad \tilde{\phi} = \tilde{\phi}(A_\mu, \phi)\,. \tag{1.3}$$

It also involves the identification of the string field Λ which corresponds to the conventional gauge transformation property of A_μ. In this paper we show that it is indeed possible to find such a relationship at the first non-linear level in the fields. To do this we use methods developed to identify general coordinate transformation and anti-symmetric tensor gauge transformation as a set of string field theoretic symmetries developed for the non-polynomial closed string field theory by [15].[1]

The fact that gauge field of the low energy effective action involves not only $\tilde{A}_\mu$ but also other fields is important in determining the coefficient of the kinetic term of A_μ from string field theory. Determining the naive kinetic term of $\tilde{A}_\mu$ is not enough. This field redefinition involved in relating A_μ and $\tilde{A}_\mu$ is also important in determining terms of the non-abelian Dirac Born Infeld action from string field theory as done in [4]. This would help in understanding the discrepancy in gauge invariance which arose in trying to determining the coefficients of certain terms in the non-abelian Dirac Born Infeld action from string field theory.

Thus it seems difficult to compare the effective action $\mathcal{S}(\tilde{A}_\mu, \tilde{\phi})$ obtained from string field theory with the world volume action of unstable branes and verify the claims of [13]. It still must be possible to see if U(1) gauge field is no longer dynamical from string field theory. To this end, it is of interest to calculate the two point functions of all vector particles at the tachyon condensate using level truncation. It is of interest to compare with the situation in p-adic string theory [14]. The kinetic terms for the translationaly zero modes of the soliton in p-adic string theory disappears after field redefinition at the new stationary point.[2]

The organization of this paper is as follows. In section 2 we review Witten's bosonic open string field theory [16] to set up our notations and conventions. We use the formulation of describing string field theory in arbitrary background field developed by [17] and [18]. Then we show the existence of the field redefinition (1.3). We also identify the set of string field theory transformations which correspond to the gauge transformation of the U(1) gauge field in the low energy effective action. These are shown to exist at the first non-linear order in the fields. In section 3 we review of how the the free action for the tachyon and the gauge field arises from string field theory. In section 4 we obtain the effective action $\mathcal{S}(\tilde{A}_\mu, \tilde{\phi})$ by eliminating all other massive and auxiliary fields. In section 5 we derive the constraint equations for field redefinition. In section 6 we write down the consistency conditions which should be identically satisfied. In section 7 we verify the consistency conditions are identically satisfied and in section 8 we show the existence of the solutions for the consistency conditions and thus prove the existence of the field redefinition up to the first non-linear order in the fields. This also identifies the string field theory transformation which corresponds to the gauge transformation of the U(1) field of the low energy effective action. In section 9 we examine the two point function of the transverse photon. We show explictly that the kinetic term for the transverse photon does

[1]The author thanks A. Sen for pointing out this reference.

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not decrease as fast as the tachyon potential approaches zero in the level truncation approximation. We show that at level $(3,6)$ there is a physical vector excitation in the spectrum, but at level $(4,8)$ this might be removed or pushed to a higher energy.

2. Review of string field theory

The open string field theory action is given by

$$\mathcal{S} = -\frac{1}{g^2}\left(\frac{1}{2}\langle I\circ\Phi(0)Q_B\Phi(0)\rangle + \frac{1}{3}\langle f_1\circ\Phi(0)f_1\circ\Phi(0)f_3\circ\Phi(0)\rangle\right) \tag{2.1}$$

g is the open string coupling constant. Q_B is the BRST charge, and by $\langle\cdot\rangle$ we mean correlation functions evaluated in the SL$(2,\mathbb{R})$ vacuum $\langle 0|\cdot|0\rangle$ Here Φ stands for the vertex operator in the conformal field theory of the first quantized open string theory with ghost number one. Using the standard state to operator mapping the field Φ creates the state $|\Phi\rangle$ out of the SL$(2,\mathbb{R})$ vacuum.

$$|\Phi\rangle = \Phi(0)|0\rangle\,. \tag{2.2}$$

As Φ has ghost number one, the Hilbert space of the first quantized open string theory including the b and c ghost fields is restricted to ghost number one. $f\circ\Phi(z)$ denotes the conformal transformation of the field Φ by the map $f(z)$. For example if Φ is a primary field of weight h then $f\circ\Phi(z) = (f'(z))^h\Phi(f(z))$. The conformal transformations f_1, f_2, f_3 and I are are given below.

$$f_1(z) = -e^{-i\pi/3}\frac{\left[\left(\frac{1-iz}{1+iz}\right)^{2/3}-1\right]}{\left[\left(\frac{1-iz}{1+iz}\right)^{2/3}+e^{i\pi/3}\right]}\,,\qquad f_2(z) = F(f_1(z))\,,$$
$$f_3(z) = F(f_2(z))\,,\qquad F(z) = -\frac{1}{1+z}\,,\qquad I(z) = -\frac{1}{z}\,. \tag{2.3}$$

It is convenient to work in a given basis of the Hilbert space of the first quantized string theory with ghost number one. Therefore we write

$$|\Phi\rangle = \sum_r \psi_r|\Phi_{1;r}\rangle \tag{2.4}$$

r denotes the various fields as well as momenta. The subscript 1 in $|\Phi_{1;r}\rangle$ stands for the ghost number. In terms of this basis the string field theory action is given by

$$\mathcal{S} = -\frac{1}{g^2}\left(\frac{1}{2}\tilde{A}^{(2)}_{rs}\psi_r\psi_s + \frac{1}{3}\tilde{A}^{(3)}_{rst}\psi_r\psi_s\psi_t\right)\,. \tag{2.5}$$

Here summation over the repeated indices is implied. $\tilde{A}^{(2)}_{rs}$ and $\tilde{A}^{(3)}_{rst}$ are given by

$$\begin{aligned}\tilde{A}^{(2)}_{rs} &= \langle I\circ\Phi_{1;r}(0)Q_B\Phi_{1,s}(0)\rangle\,,\\ \tilde{A}^{(3)}_{rst} &= \langle f_1\circ\Phi_{1;r}(0)f_2\circ\Phi_{1,s}(0)f_3\circ\Phi_{1,t}(0)\rangle\,.\end{aligned} \tag{2.6}$$

The vertices $\tilde{A}^{(2)}_{rs}$ and $\tilde{A}^{(3)}_{rst}$ have the following symmetry property

$$\tilde{A}^{(2)}_{rs} = \tilde{A}^{(2)}_{sr}\,, \qquad \tilde{A}^{(3)}_{rst} = \tilde{A}^{(3)}_{str} = \tilde{A}^{(3)}_{trs}\,. \tag{2.7}$$

The string field theory action has a large gauge symmetry. The infinitesimal gauge transformation law of ψ_s is given by

$$\delta\psi_r = A^{(2)}_{r\alpha}\lambda_\alpha + A^{(3)}_{r\alpha s}\lambda_\alpha\psi_s\,, \tag{2.8}$$

where

$$\begin{aligned} A^{(2)}_{r\alpha} &= \langle\Phi^c_{2;r}|Q_B|\Phi_{0;\alpha}\rangle\,, \\ A^{(3)}_{r\alpha s} &= \Big(\langle f_1\circ\Phi^c_{2;r}(0) f_2\circ\Phi_{1;s}(0) f_3\circ\Phi_{0;\alpha}(0)\rangle - \\ &\quad - \langle f_1\circ\Phi^c_{2;r}(0) f_2\circ\Phi_{0;\alpha}(0) f_3\circ\Phi_{1;s}(0)\rangle\Big)(-1)^{n_r+n^g_r}\,, \end{aligned} \tag{2.9}$$

where n_r and n^g_r denotes the fact that $|\Phi_{1;r}\rangle$ is an n_rth level secondary in the matter sector, and n^g_rth level secondary in the ghost sector. $\langle\Phi^c_{2;r}|$ is conjugate to the state $|\Phi_{1;r}\rangle$, that is

$$\langle\Phi^c_{2;r}|\Phi_{1;s}\rangle = \delta_{rs}\,. \tag{2.10}$$

In addition to the gauge symmetries, the string field theory action has a "trivial" symmetry of the form

$$\delta\psi_r = K_{rs}(\{\psi\})\frac{\delta\mathcal{S}}{\delta\psi_s}\,, \tag{2.11}$$

where K_{rs} is an antisymmetric in r and s and is a function of the components ψ_r. It is easy to verify that this is a symmetry of the action. This symmetry will play an important role in the field redefinition.

3. Free action of the tachyon and the gauge field

In this section we review how to obtain the free action of the tachyon and the gauge field as a warm up exercise for the next sections. We work out the action for the first two levels in the expansion of Φ.

$$|\Phi\rangle = \int dk\tilde{\phi}(k)c_1|k\rangle + \int dk\tilde{A}_\mu(k)c_1\alpha^\mu_{-1}(0)|k\rangle + \int dkF(k)c_0|k\rangle\,. \tag{3.1}$$

Here $\tilde{\phi}$ corresponds to the tachyon, $\tilde{A}_\mu$ the U(1) gauge field. We will see below that $F(k)$ an auxiliary field. By $\int dk$ we mean the $\int d^{26}k$, the state $|k\rangle$ denotes $e^{ikX(0)}|0\rangle$. Substituting this expansion in (2.5) and keeping only the quadratic vertices we obtain the following action for the first two levels:

$$\begin{aligned} \mathcal{S} = -\frac{1}{g^2}\bigg[\int dk\,\Big(&\tilde{\phi}(-k)(2k^2-1)\tilde{\phi}(k) + \tilde{A}_\mu(-k)(2k^2)\tilde{A}^\mu(k) - \\ &- 2F(-k)F(k) + 4k^\mu\tilde{A}_\mu(-k)F(k)\Big)\bigg]\,. \end{aligned} \tag{3.2}$$

We are working in the units $\alpha' = 2$. The gauge transformation of these fields can be determined by using (2.8). Restricting to only the linearized gauge transformation that is, only terms from $A^{(2)}_{r\alpha}$ in (2.8) we obtain

$$\delta\tilde{\phi}(k) = 0\,, \qquad \delta\tilde{A}_\mu(k) = -2k_\mu\lambda(k)\,, \qquad \delta F(k) = 2k^2\lambda(k)\,. \tag{3.3}$$

Here $\lambda(k)$ is the infinitesimal gauge parameter. The quadratic piece of the action in (2.5) is invariant under linearized gauge transformation. It is easy to verify the action in (3.2) is invariant under the transformation of (3.3)

The auxiliary field $F(k)$ can be eliminated using the equations of motion

$$F(k) = -k^\mu \tilde{A}_\mu(k)\,. \tag{3.4}$$

On eliminating $F(k)$, the action reduces to that of the tachyon with the standard kinetic term for the U(1) gauge field. It is given by

$$\begin{aligned}\mathcal{S} = -\frac{1}{g^2}\Bigg[\int dk\tilde{\phi}(-k)(2k^2-1)\tilde{\phi}(k) + \tilde{A}_\mu(-k)(2k^2)\tilde{A}^\mu(k) - \\ - 2\tilde{A}_\mu(-k)k^\mu k^\nu \tilde{A}_\nu(k)\Bigg]\,.\end{aligned} \tag{3.5}$$

Let us now define the gauge parameter $i\eta(k) = 2\lambda(k)$, the gauge transformation of the U(1) field $\tilde{A}_\mu$(k) is given by

$$\delta\tilde{A}_\mu(k) = ik_\mu\eta(k)\,. \tag{3.6}$$

Now it is clear that at the free level the tachyon $\tilde{\phi}$ and the gauge field $\tilde{A}_\mu(k)$ can be identified with the tachyon $\phi(k)$ and the gauge field $A_\mu(k)$ of the low energy effective action.

4. Elimination of the massive and auxiliary fields

Let us first obtain the low energy effective action in terms of the tachyon $\tilde{\phi}$ and the U(1) gauge field $\tilde{A}_\mu$. We do this by eliminating all other the massive fields and auxiliary fields in the string field Φ using their equations of motion. Of course such an effective action is valid only to tree level.

We divide the string fields ψ_r into two kinds. $\psi_r, \psi_s, \ldots$ stand for the tachyon $\tilde{\phi}(k)$ and $\tilde{A}_\mu(k)$ including their momentum index. The fields $\psi_a, \psi_b, \ldots$ stand for higher string modes, auxiliary fields and their momenta. Solving for ψ_a in terms of ψ_r up to the first non-linear order in the fields we obtain

$$\psi_a = E^{(0)}_{ar}\psi_r + \frac{1}{2}E^{(1)}_{ars}\psi_r\psi_s + \mathrm{O}(\psi^3)\,, \tag{4.1}$$

where $E^{(0)}_{ar}$ and $E^{(1)}_{ars}$ are determined from the equation of motion for ψ_a. $E^{(1)}_{ars}$ is symmetric in r and s, by definition. They are given by solving the following equations:

$$\tilde{A}^{(2)}_{ar} + \tilde{A}^{(2)}_{ab} E^{(0)}_{br} = 0\,,$$
$$\frac{1}{2}\tilde{A}^{(2)}_{ab} E^{(1)}_{brs} + \tilde{A}^{(3)}_{ars} + \tilde{A}^{(3)}_{abs} E^{(0)}_{br} + \tilde{A}^{(3)}_{arb} E^{(0)}_{bs} + \tilde{A}^{(3)}_{abc} E^{(0)}_{br} E^{(0)}_{cs} = 0\,. \qquad (4.2)$$

To solve these equations we need $\tilde{A}^{(2)}_{ab}$ to be invertible. The zero eigen modes of this operator arises from BRST exact states. So we restrict ourselves to states which are not BRST exact to ensure that the operator $\tilde{A}^{(2)}_{ab}$ is invertible. The linearized equation of motion for ψ_r now becomes

$$\tilde{A}^{(2)}_{sr}\psi_r + \tilde{A}^{(2)}_{sa} E^{(0)}_{ar}\psi_r = 0\,. \qquad (4.3)$$

From the gauge transformation of ψ_a at the linear order we obtain the relation

$$A^{(2)}_{a\alpha} = E^{(0)}_{ar} A^{(2)}_{r\alpha}\,. \qquad (4.4)$$

The gauge transformation of ψ_r up to the first nonlinear order can be written down as

$$\delta\psi_r = A^{(2)}_{r\alpha}\lambda_\alpha + \left(A^{(3)}_{r\alpha s} + A^{(3)}_{r\alpha a} E^{(0)}_{as}\right)\lambda_\alpha\psi_s + \mathrm{O}(\psi^2) \qquad (4.5)$$

It is clear from this gauge transformation, that ψ_r does not have the same gauge transformation property of the U(1) gauge field or that of the tachyon, because of the non-linear terms. The conventional low energy effective action of open bosonic string theory has the tachyon denoted by $\phi(k)$ and the gauge field, $A_\mu(k)$, The action should be invariant under the following transformation

$$\delta\phi(k) = 0\,, \qquad \delta A_\mu = ik_\mu\eta(k)\,, \qquad (4.6)$$

where $\eta(k)$ is the gauge transformation parameter. Let us call these low energy fields as ϕ_i. The index i labels the tachyon, the gauge field and their momenta. Let the gauge parameters of these fields be labelled by η_κ. It must be possible to relate ψ_r to ϕ_i so that ϕ_i has the gauge transformation given by (4.6). It also must be possible to relate the U(1) gauge invariance of the low energy action to a a string field theory gauge transformation. We show that this is possible in the next section. We use the method developed by [15] for field redefinition in the closed non-polynomial string field theory. We account for the elimination of all the massive and auxiliary fields and the fact that we are dealing with open string field theory. We verify the consistency conditions for field redefinitions in open string field theory.

5. Field redefinition

In this section we find the conditions which need to be satisfied to redefine the fields that appear in the string field theory action to that which appear in the low energy effective action. Let us relate ψ_r and ϕ_i by the following general formula up to the first non linear order

$$\psi_r = C^{(0)}_{ri}\phi_i + \frac{1}{2}C^{(1)}_{rij}\phi_i\phi_j + \mathrm{O}(\phi^3)\,. \tag{5.1}$$

The gauge parameter λ_α is related to the gauge parameters of the low energy fields by the following general formula

$$\lambda_\alpha = D^{(0)}_{\alpha\kappa}\eta_\kappa + D^{(1)}_{\alpha\kappa i}\eta_\kappa\phi_i + \mathrm{O}(\phi)\,. \tag{5.2}$$

In addition to the gauge symmetry of the string field theory action, the action is also invariant under the transformation

$$\begin{aligned}\delta\psi_r &= K_{rs}\frac{\delta\mathcal{S}}{\delta\psi_s} + K_{ra}\frac{\delta\mathcal{S}}{\delta\psi_a}\\ &= K_{rs}\left(\tilde{A}^{(2)}_{st}\psi_t + \tilde{A}^{(2)}_{sa}E^{(0)}_{at}\psi_t\right) + \mathrm{O}(\psi^2)\,.\end{aligned} \tag{5.3}$$

Where we have used the fact that we have eliminated the fields ψ_a using the equation of motion. We have kept terms only up to linear order in the fields. We now assume the most general form for the function K_{rs}

$$K_{rs} = K^{(1)}_{rs\kappa}\eta_\kappa + \mathrm{O}(\phi)\,. \tag{5.4}$$

The low energy fields ϕ_i have the following gauge transformation property

$$\delta\phi_i = B^{(2)}_{i\kappa}\eta_\kappa\,. \tag{5.5}$$

Substituting (5.5) in the variation obtained from (5.1) we get

$$\delta\psi_r = C^{(0)}_{ri}B^{(2)}_{i\kappa}\eta_\kappa + C^{(1)}_{rij}\phi_i B^{(2)}_{j\kappa}\eta_\kappa\,. \tag{5.6}$$

Now from (4.5), (2.8) , (5.3), (5.4) we obtain

$$\begin{aligned}\delta\psi_r =\; & A^{(2)}_{r\alpha}D^{(0)}_{\alpha\kappa}\eta_\kappa + A^{(2)}_{r\alpha}D^{(1)}_{\alpha\kappa i}\eta_\kappa\phi_i + \left(A^{(3)}_{r\alpha s} + A^{(3)}_{r\alpha a}E^{(0)}_{as}\right)D^{(0)}_{\alpha\kappa}C^{(0)}_{si}\eta_\kappa\phi_i +\\ & + K^{(1)}_{rs\kappa}\left(\tilde{A}^{(2)}_{st} + \tilde{A}^{(2)}_{sa}E^{(0)}_{at}\right)C^{(0)}_{ti}\eta_\kappa\phi_i\,.\end{aligned} \tag{5.7}$$

From comparing (5.6) and (5.7) we get the following set of equations

$$C^{(0)}_{ri}B^{(2)}_{i\kappa}\eta^{(\rho)}_\kappa = A^{(2)}_{r\alpha}D^{(0)}_{\alpha\kappa}\eta^{(\rho)}_\kappa\,, \tag{5.8}$$

$$\begin{aligned}C^{(1)}_{rij}\phi^{(m)}_i B^{(2)}_{j\kappa}\eta^{(\rho)}_\kappa =\; & A^{(2)}_{r\alpha}D^{(1)}_{\alpha\kappa i}\eta^{(\rho)}_\kappa\phi^{(m)}_i + \left(A^{(3)}_{r\alpha s} + A^{(3)}_{r\alpha a}E^{(0)}_{as}\right)D^{(0)}_{\alpha\kappa}C^{(0)}_{si}\eta^{(\rho)}_\kappa\phi^{(m)}_i +\\ & + K^{(1)}_{rs\kappa}\left(\tilde{A}^{(2)}_{st} + \tilde{A}^{(2)}_{sa}E^{(0)}_{at}\right)C^{(0)}_{ti}\eta^{(\rho)}_\kappa\phi^{(m)}_i\,.\end{aligned} \tag{5.9}$$

Where we have introduced a complete set of gauge transformations $\{\eta^{(\rho)}_\kappa\}$ and a complete set of field configurations $\{\phi^{(m)}_i\}$ We have already seen in the previous section that we are able to find $C^{(0)}_{ri}$ and $D^{(0)}_{\alpha\kappa}$ such that (5.8) is satisfied. It remains to be seen that if one can find $C^{(1)}_{rij}$, $D^{(1)}_{\alpha\kappa i}$ and $K^{(1)}_{rs\kappa}$ such that (5.9) can be satisfied. It is easy to extend these constraint equations to higher orders in fields. There are obstructions to this. These obstructions can arise if under some conditions (5.9) reduces to equations involving known quantities which should be satisfied identically. In the next section we will enumerate them.

6. Consistency conditions for field redefinition

There are three consistency conditions in all for the (5.9). We will enumerate each of them in the following subsections. These are similar to the consistency conditions found in [15] for the closed string. The difference being that we have eliminated all the auxiliary and massive fields using equations of motion and we are dealing with open string field theory.

6.1 Condition A

Divide the complete set of fields $\{\phi_i^{(m)}\}$ to 2 sets $\{\hat{\phi}_i^{(\hat{m})}\}$ and $\{\bar{\phi}_i^{(\bar{m})}\}$. The set $\{\hat{\phi}_i^{(\hat{m})}\}$ satisfies linearized equation of motion and the set $\{\bar{\phi}_i^{(\bar{m})}\}$ does not.

$$\begin{aligned} \left(\tilde{A}^{(2)}_{st} + \tilde{A}^{(2)}_{sa} E^{(0)}_{at}\right) C^{(0)}_{ti} \hat{\phi}_i^{(\hat{m})} &= 0\,, \quad \text{for all } s\,, \\ \left(\tilde{A}^{(2)}_{st} + \tilde{A}^{(2)}_{sa} E^{(0)}_{at}\right) C^{(0)}_{ti} \bar{\phi}_i^{(\bar{m})} &\neq 0\,, \quad \text{for some } s\,. \end{aligned} \tag{6.1}$$

Now divide the complete set of gauge transformations $\{\eta_\kappa^{(\rho)}\}$ to sets $\{\hat{\eta}_\kappa^{(\hat{\rho})}\}$ and $\{\bar{\eta}_\kappa^{(\bar{\rho})}\}$ such that

$$B^{(2)}_{i\kappa} \hat{\eta}_\kappa^{(\hat{\rho})} = 0\,, \quad \text{for all } i\,, \qquad B^{(2)}_{i\kappa} \bar{\eta}_\kappa^{(\bar{\rho})} \neq 0\,, \quad \text{for some } i\,. \tag{6.2}$$

Thus $\{\hat{\eta}_\kappa^{(\hat{\rho})}\}$ are those gauge transformations for which there is no variation in the fields ϕ_i. The index r is divided also to two sets into BRST invariant and non-invariant states given by

$$\langle \hat{\Phi}^c_{2;r} | Q_B = 0\,, \qquad \langle \bar{\Phi}^c_{2;\bar{r}} | Q_B \neq 0\,. \tag{6.3}$$

Now restrict the indices r, m, ρ in (5.9) to $\hat{r}, \hat{m}$ and $\hat{\rho}$. The terms involving $C^{(1)}$ and $K^{(1)}$ drop out because of (6.1) and (6.2), respectively. The term involving $D^{(1)}$ drops off because of

$$A^{(2)}_{\hat{r}\alpha} = 0\,. \tag{6.4}$$

This follows from the definition of $A^{(2)}$ in (2.8) and (6.3). Thus we obtain the consistency condition

$$\left(A^{(3)}_{r\alpha s} + A^{(3)}_{r\alpha a} E^{(0)}_{as}\right) D^{(0)}_{\alpha\kappa} C^{(0)}_{si} \hat{\eta}_\kappa^{(\hat{\rho})} \hat{\phi}_i^{(\hat{m})} = 0\,. \tag{6.5}$$

This involves all known coefficients and should be satisfied identically.

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6.2 Condition B

The next obstruction arises because of $C^{(1)}_{rij}$ is symmetric in i and j. Restrict the index r in (5.9) to be $\hat{r}$. The term involving $D^{(1)}$ drops out because of (6.4). Take $\phi^{(m)}_j$ to be of the form $B^{(2)}_{j\kappa'}\eta^{(\rho')}_{\kappa'}$. Then term involving $K^{(1)}$ is given by

$$\begin{aligned}K^{(1)}_{\hat{r}s\kappa}\left(\tilde{A}^{(2)}_{st}+\tilde{A}^{(2)}_{sa}E^{(0)}_{at}\right)C^{(0)}_{ti}\eta^{(\rho)}_{\kappa}B^{(2)}_{j\kappa'}\eta^{(\rho')}_{\kappa'} &= K^{(1)}_{\hat{r}s\kappa}\left(\tilde{A}^{(2)}_{st}+\tilde{A}^{(2)}_{sa}E^{(0)}_{at}\right)\eta^{(\rho)}_{\kappa}A^{(2)}_{t\alpha}D^{(0)}_{\alpha\kappa'}\eta^{(\rho')}_{\kappa'}\\ &= K^{(1)}_{\hat{r}s\kappa}\left(\tilde{A}^{(2)}_{st}A^{(2)}_{t\alpha}+\tilde{A}^{(2)}_{sa}A^{(2)}_{a\alpha}\right)\eta^{(\rho)}_{\kappa}A^{(2)}_{t\alpha}D^{(0)}_{\alpha\kappa'}\eta^{(\rho')}_{\kappa'}\\ &= 0\,. \end{aligned}\tag{6.6}$$

Where we have used (5.8) in the first step and (4.4) in the second step. We have

$$\tilde{A}^{(2)}_{st}A^{(2)}_{t\alpha}+\tilde{A}^{(2)}_{sa}A^{(2)}_{a\alpha}=\langle I\circ\Phi_{1;s}Q^2_B\Phi_{0;\alpha}\rangle=0\,. \tag{6.7}$$

To arrive at the above relation we have used completeness and $Q^2_B=0$. We also use the fact correlations vanish except when saturated by fields with total ghost number three. The term involving $K^{(1)}$ also vanishes. Thus for this case from (5.9) we are left with

$$C^{(1)}_{\hat{r}ij}B^{(2)}_{i\kappa'}\eta^{(\rho')}_{\kappa'}B^{(2)}_{j\kappa}\eta^{(\rho)}_{\kappa}=\left(A^{(3)}_{\hat{r}\alpha s}+A^{(3)}_{\hat{r}\alpha a}E^{(0)}_{as}\right)D^{(0)}_{\alpha\kappa}C^{(0)}_{si}B^{(2)}_{i\kappa'}\eta^{(\rho)}_{\kappa}\eta^{(\rho')}_{\kappa'}\,. \tag{6.8}$$

As $C^{(1)}_{\hat{r}ij}$ is symmetric in i and j we have the following constraint:

$$\left(A^{(3)}_{\hat{r}\alpha s}+A^{(3)}_{\hat{r}\alpha a}E^{(0)}_{as}\right)D^{(0)}_{\alpha\kappa}C^{(0)}_{si}B^{(2)}_{i\kappa'}\left(\eta^{(\rho)}_{\kappa}\eta^{(\rho')}_{\kappa'}-\eta^{(\rho')}_{\kappa'}\eta^{(\rho)}_{\kappa}\right)=0\,. \tag{6.9}$$

6.3 Condition C

We now find the third and final obstruction. This is due to the antisymmetry of $K^{(1)}_{rs\alpha}$ in r and s. In (5.8) choose the index ρ to be $\hat{\rho}$. Then by (6.2) the term involving $C^{(1)}$ in (5.8) drops out. Now multiply the equation by $(\tilde{A}^{(2)}_{rt'}+E^{(0)}_{ar}\tilde{A}^{(2)}_{at'})C^{(0)}_{t'j}\phi^{(m')}_{j'}$ The term involving $D^{(1)}$ is then given by

$$\begin{aligned}A^{(2)}_{r\alpha}\left(\tilde{A}^{(2)}_{rt'}+E^{(0)}_{ar}\tilde{A}^{(2)}_{at'}\right)C^{(0)}_{t'j'}\phi^{(m')}_{j'}D^{(1)}_{\alpha\kappa i}\hat{\eta}^{(\hat{\rho})}_{\kappa}\phi^{(m)}_{i} &= \left(A^{(2)}_{r\alpha}\tilde{A}^{(2)}_{rt'}+A^{(2)}_{a\alpha}\tilde{A}^{(2)}_{at'}\right)\times\\ &\quad\times C^{(0)}_{t'j'}\phi^{(m')}_{j'}D^{(1)}_{\alpha\kappa i}\hat{\eta}^{(\hat{\rho})}_{\kappa}\phi^{(m)}_{i}\,. \end{aligned}\tag{6.10}$$

Where we have used (4.4). Now using the fact that $\tilde{A}^{(2)}$ is a symmetric matrix and (6.7), the term involving $D^{(1)}$ vanishes. Thus (5.9) becomes

$$\begin{aligned}&\left(A^{(3)}_{r\alpha s}+A^{(3)}_{r\alpha a}E^{(0)}_{as}\right)\left(\tilde{A}^{(2)}_{rt'}+E^{(0)}_{ar}\tilde{A}^{(2)}_{at'}\right)C^{(0)}_{t'j'}D^{(0)}_{\alpha\kappa}C^{(0)}_{si}\hat{\eta}^{(\hat{\rho})}_{\kappa}\phi^{(m')}_{j'}\phi^{(m)}_{i}+\\ &+K^{(1)}_{rs\kappa}\left(\tilde{A}^{(2)}_{st}+\tilde{A}^{(2)}_{sa}E^{(0)}_{at}\right)C^{(0)}_{ti}\hat{\eta}^{(\hat{\rho})}_{\kappa}\phi^{(m)}_{i}\left(\tilde{A}^{(2)}_{rt'}+\tilde{A}^{(2)}_{rb}E^{(0)}_{bt'}\right)C^{(0)}_{t'j}\hat{\eta}^{(\hat{\rho}')}_{\kappa'}\phi^{(m')}_{i'}=0\,. \end{aligned}\tag{6.11}$$

As $K^{(1)}_{rs\alpha}$ is antisymmetric in r and s, the symmetric component in r and s should be zero. Therefore we have the following constraint

$$\begin{aligned}&\left(A^{(3)}_{r\alpha s}+A^{(3)}_{r\alpha a}E^{(0)}_{as}\right)\left(\tilde{A}^{(2)}_{rt'}+E^{(0)}_{ar}\tilde{A}^{(2)}_{at'}\right)C^{(0)}_{t'j'}D^{(0)}_{\alpha\kappa}C^{(0)}_{si}\hat{\eta}^{(\hat{\rho})}_{\kappa}\times\\ &\qquad\times\left(\phi^{(m')}_{j'}\phi^{(m)}_{i}+\phi^{(m)}_{j'}\phi^{(m')}_{i}\right)=0\,. \end{aligned}\tag{6.12}$$

7. Verification of the consistency conditions

In this section we show that each of the consistency conditions found in the previous section is satisfied identically for the open superstring theory.

7.1 Condition A

Let us first verify that equation (6.5) is satisfied. Define

$$\left|\hat{\Psi}_1^{(\hat{m})}\right\rangle = C^{(0)}_{si}\hat{\phi}^{(\hat{m})}_i|\Phi_{1;s}\rangle + E^{(0)}_{as}C^{(0)}_{si}\hat{\phi}^{(\hat{m})}_i|\Phi_{1;a}\rangle\,. \tag{7.1}$$

Now

$$\langle\Phi_{1;r}|Q_B\left|\hat{\Psi}^{(\hat{m})}\right\rangle = C^{(0)}_{si}\hat{\phi}^{(\hat{m})}_i\left(\tilde{A}^{(2)}_{rs} + \tilde{A}^{(2)}_{ra}E^{(0)}_{as}\right) = 0\,. \tag{7.2}$$

Where we have used (6.1). Also from (4.2) we have

$$\langle\Phi_{1;a}|Q_B\left|\hat{\Psi}^{(\hat{m})}\right\rangle = C^{(0)}_{si}\hat{\phi}^{(\hat{m})}_i\left(\tilde{A}^{(2)}_{as} + \tilde{A}^{(2)}_{ab}E^{(0)}_{bs}\right) = 0\,. \tag{7.3}$$

This implies that $|\hat{\Psi}^{(\hat{m})}\rangle$ is a BRST invariant state.

$$Q_B\left|\hat{\Psi}^{(\hat{m})}\right\rangle = 0\,. \tag{7.4}$$

Now define

$$\left|\hat{\Lambda}^{(\hat{\rho})}\right\rangle = D^{(0)}_{\alpha\kappa}\hat{\eta}^{(\hat{\rho})}_\kappa|\Phi_{0;\alpha}\rangle\,. \tag{7.5}$$

We use (5.8) and (6.2) to show

$$\langle\Phi^c_{2;r}|Q_B\left|\hat{\Lambda}^{(\hat{\rho})}\right\rangle = D^{(0)}_{\alpha\kappa}\hat{\eta}^{(\hat{\rho})}_\kappa\langle\Phi^c_{2;r}|Q_B|\Phi_{0\alpha}\rangle = D^{(0)}_{\alpha\kappa}\hat{\eta}^{(\hat{\rho})}_\kappa A^{(2)}_{r\alpha} = C^{(0)}_{ri}B^{(2)}_{i\kappa}\hat{\eta}^{(\hat{\rho})}_\kappa = 0\,. \tag{7.6}$$

Also we have

$$\begin{aligned}\langle\Phi^c_{2;a}|Q_B\left|\hat{\Lambda}^{(\hat{\rho})}\right\rangle &= D^{(0)}_{\alpha\kappa}\hat{\eta}^{(\hat{\rho})}_\kappa\langle\Phi^c_{2;a}|Q_B|\Phi_{0\alpha}\rangle = D^{(0)}_{\alpha\kappa}\hat{\eta}^{(\hat{\rho})}_\kappa A^{(2)}_{a\alpha}\\ &= C^{(0)}_{ri}B^{(2)}_{i\kappa}\hat{\eta}^{(\hat{\rho})}_\kappa E^{(0)}_{ra} = 0\,.\end{aligned} \tag{7.7}$$

Where we have used (4.4) and (6.2). From (7.6) and (7.7) we obtain that $|\hat{\Lambda}^{(\hat{\rho})}\rangle$ is a BRST invariant state.

$$Q_B\left|\hat{\Lambda}^{(\hat{\rho})}\right\rangle = 0\,. \tag{7.8}$$

Using definitions (7.1), (7.5) and (2.9) the consistency condition (6.5) reduces to

$$\left\langle f_1\circ\hat{\Phi}^c_{\hat{r}} f_2\circ\hat{\Lambda}^{(\hat{\rho})} f_3\circ\hat{\Psi}^{(\hat{m})}\right\rangle - \left\langle f_1\circ\hat{\Phi}^c_{\hat{r}} f_2\circ\hat{\Psi}^{(\hat{m})} f_3\circ\hat{\Lambda}^{(\hat{\rho})}\right\rangle = 0\,. \tag{7.9}$$

As all the field in (7.9) are BRST invariant fields, if any of them is BRST exact, then the equation is automatically satisfied. Therefore we have to look for BRST invariant but not BRST exact fields. The linearized gauge transformation parameter for the tachyon field is zero. Thus by standard BRST analysis the only gauge transformation we have is that for the photon. This is given by $|\hat{\Lambda}^{(\hat{\rho})}\rangle = \xi|0\rangle$ The ghost number one fields in the formula (7.9) can by the on shell tachyon $|\hat{\Psi}^{(\hat{m})}\rangle = c_1|k\rangle$ with $k^2 = 1/2$, the mass shell condition. Or it is the on shell gauge field given by $|\hat{\Psi}^{(\hat{m})}\rangle = \epsilon_\mu c_1 \alpha^\mu_{-1}|k\rangle$ with $k^\mu \epsilon_\mu = 0$ and $k^2 = 0$. The conjugate fields in (7.9) are the fields conjugate to the on shell tachyon and the on shell gauge field. We can have $\langle \Phi^c_{\hat{r}}| = \langle k|c_{-1}c_0$ with $k^2 = 1/2$, or we have $\langle \Phi^c_{\hat{r}}| = \langle k|c_{-1}c_0\alpha^\mu_{-1}a_\mu$ with $a_\mu k^\mu = 0$ and $k^2 = 0$. Evaluating the l.h.s. of (7.9) with any of these states it can be seen that it is zero. Thus the consistency condition (6.5) is identically satisfied.

7.2 Condition B

Now let us examine the obstruction(6.9). Define $|\Lambda^{(\rho)}\rangle = D^{(0)}_{\alpha\kappa}\eta^{(\rho)}_\kappa|\Phi_{0;\alpha}\rangle$. Then we have

$$\begin{aligned} C^{(0)}_{si}B^{(2)}_{i\kappa'}\eta^{(\rho')}_{\kappa'}|\Phi_{1;s}\rangle &+ E^{(0)}_{as}C^{(0)}_{si}B^{(2)}_{i\kappa'}\eta^{(\kappa')}_{\rho'}|\Phi_{1;a}\rangle = \\ &= A^{(2)}_{s\alpha}D^{(0)}_{\alpha\kappa'}\eta^{(\rho')}_{\kappa'}|\Phi_{1;s}\rangle + E^{(0)}_{as}A^{(2)}_{s\alpha}D^{(0)}_{\alpha\kappa'}\eta^{(\rho')}_{\kappa'}|\Phi_{1;a}\rangle \\ &= \langle\Phi^c_{2;s}|Q_B|\Phi_{0;\alpha}\rangle D^{(0)}_{\alpha\kappa'}\eta^{(\rho')}_{\kappa'}|\Phi_{1;s}\rangle + \langle\Phi^c_{2;a}|Q_B|\Phi_{0\alpha}\rangle D^{(0)}_{\alpha\kappa'}\eta^{(\rho')}_{\kappa'}|\Phi_{1;a}\rangle \\ &= D^{(0)}_{\alpha\kappa'}\eta^{(\rho')}_{\kappa'}Q_B|\Phi_{0;\alpha}\rangle = Q_B|\Lambda^{(\rho')}\rangle\,. \end{aligned} \tag{7.10}$$

Here we have used (5.8), (2.9) and completeness. Thus the consistency condition (6.9) can be written as

$$\begin{aligned} \Big\langle f_1\circ\Phi^c_{\hat{r}}(0) f_2\circ\Lambda^{(\rho)}(0) f_3\circ Q_B\Lambda^{(\rho')}(0)\Big\rangle - \Big\langle f_1\circ\Phi^c_{\hat{r}}(0) f_2\circ Q_B\Lambda^{(\rho')}(0) f_3\Lambda^{(\rho)}(0)\Big\rangle - \\ -(\rho\leftrightarrow\rho')\,. \end{aligned} \tag{7.11}$$

Now, one can deform the contour of Q_B in the first set of term so that it will act on Λ^ρ. This is because $|\Phi^c_{\hat{r}}\rangle$ is BRST invariant. Then, the first set of term cancel against the second set of terms. Thus the consistency condition (6.9) is satisfied identically.

7.3 Condition C

Finally we analyze the third consistency condition (6.12). Define

$$|\Psi^{(m)}\rangle = C^{(0)}_{tj}\phi^{(m)}_j|\Phi_{1,t}\rangle + E^{(0)}_{at}C^{(0)}_{tj}\phi^{(m)}_j|\Phi_{1,a}\rangle = \psi^{(m)}_t|\Psi_{1;t}\rangle + \psi^{(m)}_a|\Psi_{1;a}\rangle \tag{7.12}$$

and $|\Lambda^\rho\rangle = D^{(0)}_{\alpha\kappa}\eta^{(\rho)}_\kappa|\Phi_{0;\alpha}\rangle$. Using these definitions (6.12) can be written as

$$\begin{aligned} A^{(3)}_{r\rho s}\tilde{A}^{(2)}_{rt'}\psi^{(m')}_{t'}\psi^{(m)}_s + A^{(3)}_{r\rho a}\tilde{A}^{(2)}_{rt'}\psi^{(m')}_{t'}\psi^{(m)}_a + A^{(3)}_{r\rho s}\tilde{A}^{(2)}_{ra'}\psi^{(m')}_{a'}\psi^{(m)}_s + \\ + A^{(3)}_{r\rho a}\tilde{A}^{(2)}_{ra'}\psi^{(m')}_{a'}\psi^{(m)}_a + (m\leftrightarrow m') = 0\,. \end{aligned} \tag{7.13}$$

Where we have used the linearized equation of motion of $\psi^{(m)}_a$ and (4.2). Now one can add to this equation the following equation:

$$\begin{aligned} A^{(3)}_{b\rho s}\tilde{A}^{(2)}_{bt'}\psi^{(m')}_{t'}\psi^{(m)}_s + A^{(3)}_{b\rho a}\tilde{A}^{(2)}_{bt'}\psi^{(m')}_{t'}\psi^{(m)}_a + A^{(3)}_{b\rho s}\tilde{A}^{(2)}_{ba'}\psi^{(m')}_{a'}\psi^{(m)}_s + \\ + A^{(3)}_{b\rho a}\tilde{A}^{(2)}_{ba'}\psi^{(m')}_{a'}\psi^{(m)}_a + (m\leftrightarrow m') = 0\,. \end{aligned} \tag{7.14}$$

This equation holds because $\psi_a^{(m)}$ satisfies linearized equation of motion.

$$\tilde{A}^{(2)}_{ar}\psi_r^{(m)} + \tilde{A}^{(2)}_{ab}\psi_b^{(m)} = 0\,. \tag{7.15}$$

We are justified in using the linear equations of motion because the terms in (6.12) are quadratic in the fields. Adding (7.13) and (7.14) and using completeness, the constraint (6.12) becomes

$$\begin{aligned}&\Big(\Big\langle f_1\circ Q_B\Psi^{(m)}(0) f_2\circ\hat{\Lambda}^{(\hat{\rho})}(0) f_3\circ\Psi^{(m')}(0)\Big\rangle -\\ &- \Big\langle f_1\circ Q_B\Psi^{(m)}(0) f_2\circ\Psi^{(m')}(0) f_3\circ\hat{\Lambda}^{(\hat{\rho})}(0)\Big\rangle\Big) + (m\leftrightarrow m') = 0\,.\end{aligned} \tag{7.16}$$

One can now deform the contour of Q_B in the first set of terms so that it acts on $|\Psi^{(m')}\rangle$. It does not act on on $|\hat{\Lambda}^{(\hat{\rho})}\rangle$ as it is BRST invariant. In deforming the contour one picks up minus sign, and another minus sign as Q_B crosses $\Psi^{(m)}$. Now these first set of terms cancel against the second set of terms. Thus (6.12) is satisfied identically.

We have shown that all the consistency conditions found in the previous section for solving (5.9) are satisfied identically.

8. Existence of field redefinition

In this section we show that there exists $C^{(1)}$ and $D^{(1)}$ which solve (5.9). This establishes that there exists a field redefinition such that in the new set of fields the tachyon and the U(1) gauge field corresponds to the fields seen in the low energy effective action to the first non-linear order in the fields. This also establishes the existence of a string gauge transformations which corresponds to the U(1) gauge transformation of the low energy effective action.

In this section we will closely follow [15]. We outline the steps involved for completeness. We divide the situations to three cases

8.1 Case A: $r\in\bar{r}$

Let us consider the case when r belongs to the type $\bar{r}$. This implies that $A^{(2)}_{\bar{r}\alpha}$ has a right inverse $M_{\alpha\bar{s}}$

$$A^{(2)}_{\bar{r}\alpha}M_{\alpha\bar{s}} = \delta_{\bar{r}\bar{s}}\,. \tag{8.1}$$

Therefore we can solve (5.9) for this case by choosing $D^{(1)}$ given by

$$\begin{aligned}D^{(1)}_{\alpha\kappa i}\eta_\kappa\phi_i = M_{\alpha\bar{r}}\Big(&C^{(1)}_{\bar{r}ij}\phi_i B^{(2)}_{j\kappa}\eta_\kappa - A^{(2)}_{\bar{r}\beta}D^{(1)}_{\beta\kappa i}\eta_\kappa\phi_i - \Big(A^{(3)}_{\bar{r}\beta s} + A^{(3)}_{r\beta a}E^{(0)}_{as}\Big) D^{(0)}_{\beta\kappa}C^{(0)}_{si}\eta_\kappa\phi_i +\\ &+ K^{(1)}_{\bar{r}s\kappa}\Big(\tilde{A}^{(2)}_{st} + \tilde{A}^{(2)}_{sa}E^{(0)}_{at}\Big) C^{(0)}_{ti}\eta^{(\rho)}_\kappa\phi^{(m)}_i\Big)\,.\end{aligned} \tag{8.2}$$

8.2 Case B: $r \in \hat{r}$, $\rho \in \bar{r}$

In this case the term involving $D^{(1)}$ in (5.9) vanishes because of (6.3). Since the index ρ is in the set $\bar{\rho}$ the matrix $S_{i\bar{\rho}} = B^{(2)}_{i\kappa}\bar{\eta}^{(\bar{\rho})}_{\kappa}$ has a left inverse $N_{\bar{\rho}j}$ which satisfies

$$N_{\bar{\rho}i}S_{i\bar{\rho}'} = \delta_{\rho\bar{\rho}'} \,. \tag{8.3}$$

We can now solve (5.9) by choosing $C^{(1)}$ given by

$$C^{(1)}_{\hat{r}ij} = \left(\left(A^{(3)}_{\hat{r}\alpha s} + A^{(3)}_{\hat{\alpha}a}E^{(0)}_{as}\right) D^{(0)}_{\alpha\kappa}C^{(0)}_{sj} + K^{(1)}_{\hat{r}s\kappa}\left(\tilde{A}^{(2)}_{st} + \tilde{A}^{(2)}_{sa}E^{(0)}_{at}\right) C^{(0)}_{tj}\right)\bar{\eta}^{(\bar{\rho})}_{\kappa}N_{\bar{\rho}i} \,. \tag{8.4}$$

It can be shown [15] that one can choose a basis for the fields $\{\phi_i\}$ such that the $C^{(1)}$ obtained from the above equation is symmetric in i and j.

8.3 Case C: $r \in \hat{r}$, $\rho \in \hat{\rho}$, $m \in \bar{m}$

The case $r \in \hat{r}$, $\rho \in \hat{\rho}$ and $m \in \hat{m}$ is already covered in (6.5). We have shown in such a case the consistency condition is satisfied identically. So, the case which remains is $r \in \hat{r}$, $\rho \in \hat{\rho}$ and $m \in \bar{m}$. In this case the terms involving $C^{(1)}$ and $D^{(1)}$ drop from (5.9) due to (6.2) and (6.3), respectively. The matrix $T_{s\bar{m}}$ given by

$$T_{s\bar{m}} = \left(\tilde{A}^{(2)}_{st} + \tilde{A}^{(2)}_{sa}E^{(0)}_{at}\right) C^{(0)}_{tj}\bar{\phi}^{(\bar{m})}_{i} \,. \tag{8.5}$$

Then the matrix $T_{s\bar{m}}$ has a left inverse $U_{\bar{m}r}$ given by

$$U_{\bar{m}r}T_{r\bar{n}} = \delta_{\bar{m}\bar{n}} \,. \tag{8.6}$$

Then we can solve (5.9) by choosing

$$K^{(1)}_{\hat{r}t\kappa}\hat{\eta}^{(\hat{\rho})}_{\kappa} = -\left(A^{(3)}_{\hat{r}\alpha s} + A^{(3)}_{\hat{\alpha}a}E^{(0)}_{as}\right) D^{(0)}_{\alpha\kappa}C^{(0)}_{sj}\hat{\eta}^{(\hat{\rho})}_{\kappa}\bar{\phi}^{(\bar{m})}_{i}U_{\bar{m}t} \,. \tag{8.7}$$

Notice that only the projection of $K^{(1)}_{\hat{r}t\kappa}$ onto the space spanned by $\hat{\eta}^{(\hat{\rho})}_{\kappa}$ is determined. The other components can be chosen to be arbitrary. It can also be shown that the $K^{(1)}_{rs\kappa}$ can be chosen in such a way that it is antisymmetric in r and s [15].

This completes the proof of the existence of field redefinition of the tachyon and the gauge field so that they can be identified with the fields that appear in the low energy effective action. This also determines a string gauge transformation, λ_α which corresponds to the U(1) gauge transformation, η_κ of the low energy fields.

9. The transverse photon at the tachyon condensate

In this section we demonstrate explictly that field redefinition is important in the relation of the Dirac-Born-Infeld action and the low energy effective action derived from Witten's String field theory. To do this we evaluate the kinetic term for the

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transverse photon A_μ^T, to level 4 in the tachyon condensate. We work in the Feynman-Siegel gauge and with $\alpha' = 1$. The quadriatic terms in the action involving the transverse photon is given by[3]

$$S = \int dk A_\mu(-k) A^\mu(k) \left(\frac{k^2}{2} + e^{-2k^2 \ln(3\sqrt{3}/4)} \mathcal{R}(k) \right). \tag{9.1}$$

Where $\mathcal{R}(k)$ is given by

$$\begin{aligned} \mathcal{R}(k) = & \frac{3\sqrt{3}}{4} t - \left(\frac{49}{12\sqrt{3}} - \frac{4}{3\sqrt{3}} k^2 \right) \frac{v}{\sqrt{13}} + \frac{11}{12\sqrt{3}} u + \\ & + \left(\frac{1579}{162\sqrt{3}} - \frac{88}{81\sqrt{3}} k^2 \right) A + \left(-\frac{539}{324\sqrt{3}} + \frac{44}{81\sqrt{3}} k^2 \right) F + \\ & + \frac{20}{27\sqrt{3}} D + \frac{19}{108\sqrt{3}} E - \frac{20}{81\sqrt{3}} C + \left(\frac{785}{54\sqrt{3}} - \frac{520}{81\sqrt{3}} k^2 + \frac{64}{81\sqrt{3}} k^4 \right) B \,. \end{aligned} \tag{9.2}$$

Here the variables $t, u, v, A, B, C, D, E, F$ stand for the fields as defined in [3]. It is now easy to extract the kinetic term. It is given by

$$S_{\text{KE}} = \int dk \mathcal{T} A_\mu(-k) k^2 A^\mu(k) \,, \tag{9.3}$$

where

$$\begin{aligned} \mathcal{T} = \frac{1}{2} - 2\ln\left(\frac{3\sqrt{3}}{4}\right) & \left(\frac{3\sqrt{3}}{4} t - \frac{49}{12\sqrt{39}} v + \frac{11}{12\sqrt{3}} u + \frac{1579}{162\sqrt{3}} A - \frac{539}{324\sqrt{3}} F + \right. \\ & \left. + \frac{20}{27\sqrt{3}} D + \frac{19}{108\sqrt{3}} E - \frac{20}{81\sqrt{3}} C + \frac{785}{54\sqrt{3}} B \right) + \\ & + \frac{4}{3\sqrt{39}} v - \frac{88}{81\sqrt{3}} A + \frac{44}{81\sqrt{3}} F - \frac{520}{81\sqrt{3}} B \,. \end{aligned} \tag{9.4}$$

In table 1 we tabulate the rate of decrease of the kinetic term of the transverse photon as we approach the tachyonic condensate by level trucation.

Level	Coefficient of k^2	% Decrease
(1,2)	.1899	37.9%
(2,4)	.1861	37.2%
(4,8)	.1814	36.3%

Table 1: Coefficient of the kinetic term.

Here the last column shows the decrease in the coefficient of the kinetic term as compared to 1/2, which is the value at the perturbative vacuum. We have used the values found in [3] for the fields t, u, v, A, B, C, D, E and F. Thus the rate of decrease of the kinetic term is much slower than the rate at which the minimum of the tachyon potential is reached in level truncation. In [13] it was shown in the Dirac-Born-Infeld action the coefficient of the kinetic term is the tachyon potential. Thus we see clearly that the effective action from Witten's open string field theory is not the Dirac-Born-Infeld action. We

[3]The action in (9.1) agrees with that of [2] (eq. (4.7)) till level 2, with $\alpha' = 1$, $g = 2$, $t = \phi$, $u = -\beta_1$, and $v = B$.

have shown that there is a field redefiniton which relates these two actions. This field redefintion is not unique. It will be interesting to find that unique field redefiniton which relates the two action explictly.

9.1 Vector excitations at the tachyon condensate

To find the physcial excitations of the transverse photon we evaluate the physical poles in the two point function of the transverse photon. This is done by evaluting the zeros of the funtion

$$f(k_0) = -\frac{k_0^2}{2} + e^{2k_0^2} \ln\left(\frac{3\sqrt{3}}{4}\right) \mathcal{R}(k_0)\,. \tag{9.5}$$

Here we have substitued $k^2 = -k_0^2$ in the two point funtion of the transverse photon from (9.1). Poles in $f(k_0)$ represent masses of physical excitations. The results are listed in table 2.

Level	Zeros of the two point function
(1,2)	No real zeros in k_0^2
(2,4)	k_0^2= 15.6031
(4,8)	k_0^2 = 13.4106

Table 2: Zeros of the two point function.

We note that the location of the pole in level 4 decreases as compared to level 2. To find the physical excitations at the tachyon condensate it is not only sufficient to look at the poles in the transverse photon two-point function. We also need to find the zeros in the determinant of the fluctuation matrix of all the vector particles. Above level $(2,4)$, vector fields at level 3 mix with the transverse photon.

We will now show that the pole found at level $(2,4)$ exists even when we consider all the vector fields at level $(3,6)$. Consider the action including all the vector fields till level 3. We write the quadriatic terms in the action as

$$\begin{aligned} S = \int dk A_\mu(-k) A^\mu(k) \left(\frac{k^2}{2} + e^{-2k^2\ln(3\sqrt{3}/4)}\mathcal{R}(k)\right) + \\ + 2A_\mu(-k)\alpha_i(k)V_\mu^i(k) + V_\mu^i(-k)M_{ij}(k)V_\mu^j(k)\,. \end{aligned} \tag{9.6}$$

Where V_μ^i are all the other vector fields till level 3. These are

$$V_\mu^1 \alpha_{-3}^\mu c_1|0\rangle + V_\mu^2 \alpha_{-1}^\mu b_{-1}c_{-1}c_1|0\rangle + V_\mu^3 \alpha_{-1}^\mu L_{-2}c_1|0\rangle\,. \tag{9.7}$$

Evaluating the zeros of the determinant of the fluctation matrix is equivalent to elimating the fields V_μ^i using their equations of motion and then evaulating the zeros of the two point funtion for the transverse photon. This is given by

$$S = \int dk A_\mu(-k) A^\mu(k) \left(\frac{k^2}{2} + e^{-2k^2\ln(3\sqrt{3}/4)}\mathcal{R}(k) - \alpha_i(k)M_{ij}^{-1}(k)\alpha_j(k)\right). \tag{9.8}$$

Now we look at the zeros of the function

$$g(k_0) = f(k_0) - h(k_0) = f(k_0) - \alpha_i(k_0)M_{ij}^{-1}(k_0)\alpha_j(k_0)\,. \tag{9.9}$$

Here $f(k_0)$ is defined in (9.5). $g(0)$ must be positive, This represents the (mass)^2 of the transverse photon. It is positive as there is no tachyonic excitations at the stable vacuum. For $k_0 \to \infty$ we have $f(k_0) \to -vk_0^2 e^{k_0^2 \ln(3\sqrt{3}/4)}$. We are looking at terms in the action till level 6. The value of v is positive at the tachyon condensate. The $f(k_0)$ tends to $-\infty$ as $k_0 \to \infty$. We examine the behaviour of $h(k_0)$ as k_0 tends to infinity. To do this we look at terms which has the highest power of momentum in $h(k_0)$. These are given by fields which have the highest power of L_{-2}. For each L_{-2} there is a power of k^2. Using this we find $h(k_0) \to v^2 k_0^4 e^{k_0^2 \ln(3\sqrt{3}/4)}/t$ as $k_0 \to \infty$. Thus $g(k_0)$ continues to be negative for large values of k_0. As $g(k_0)$ is positive at $k_0 = 0$, there exists at least a single zero by continuity. Therefore the pole found at level $(2,4)$ persists at level $(3,6)$.

We consider the case of level $(4,8)$. Again $g(0)$ is positive as it represents the (mass)^2 of the transverse photon. For $k_0 \to \infty$, we see that $f(k_0) \to Bk_0^4 e^{k_0^2 \ln(3\sqrt{3}/4)}$ and $h(k_0) \to -B^2 k_0^6 e^{k_0^2 \ln(3\sqrt{3}/4)}/v$. Thus $g(k_0)$ is positive for large values of k_0. Therefore this analysis is not conclusive. But, it does suggests that the zero could be removed or might be shifted to a higher value than given in (2). We also note that this method of analysis of the zeros using momentum dependence simplifies the task of finding zeros. We need only to look at a subset of correlation functions.

10. Conclusions

We have seen that the naive low energy effective action $\mathcal{S}(\tilde{A}_\mu, \tilde{\phi})$ does not correspond to the world volume action of unstable branes. To obtain the world volume action from string field theory, one has to redefine the tachyon and the gauge field. We have shown that this is possible to the first non-linear order in the fields. We have also identified a string field theory gauge symmetry which corresponds to the U(1) gauge transformation of the gauge field which appears in the low energy effective action. These considerations help in understanding the discrepancy of gauge invariance obtained for certain terms in the non-abelian Dirac Born Infeld action of [4]. It is easy to see that the low energy effective action on unstable branes contains not only the trivial U(1) gauge symmetry, but a large gauge symmetry arising from the large gauge symmetry of string field theory. It would be interesting to understand this symmetry further.

We examined the vector excitations till level $(4,8)$. We showed that there is a physical vector excitation at level $(3,6)$. At level $(4,8)$ our methods are not conclusive. But we see that this excitation can be removed or pushed to a higher energy.

Acknowledgments

The author thanks A. Sen for useful correspondence and for an initiation into this subject. He thanks N. Itzhaki for discussions and participation at the early stage of this project. He acknowledges useful discussions with D. Gross, V. Periwal and M. Srednicki. He thanks the participants of the Santa Barbara post-doc journal club for discussions. The work of the author is supported by NSF grant PHY97-22022.

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